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An Agent-based Model for Quantitatively Analyzing and Predicting the Complex Behavior of Emergency Departments

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Abstract

Hospital based emergency departments (EDs) are highly integrated service units devoted primarily to handling the needs of patients arriving without prior appointment, and with uncertain conditions. In this context, analysis and management of patient flows play a key role in developing policies and decisions for overall performance improvement. However, patient flows in EDs are considered to be very complex because of the different pathways patients may take and the inherent uncertainty and variability of healthcare processes. The agent-based model provides a flexible platform for studying ED operations, as it predicts the system-level behavior from individual level interactions. In this way, policies such as staffing can be changed and the effect on system performance, such as waiting times and throughput, can be quantified. The overall goal of this study is to develop tools to better understand the complexity, evaluate policy and improve efficiencies of ED units. The main contribution of this paper includes: an agent-based model of ED, a flexible atomic data monitoring layer for agent state tracing, and a master/worker based framework for efficiently executing the model and analyzing simulation data. The presented model has been calibrated to imitate a real ED in Spain, the simulation results have proven the feasibility of using agent-based model to study ED system.

Keywords: Emergency Department, Agent-Based Model, Complex Adaptive System, Decision Support System

1. Introduction and related work

1.1. Introduction and motivation

Nowadays, many of the healthcare systems are large, dynamic, complex environments, especially Emergency Departments (EDs). These EDs serve as the primary gateway to the acute healthcare system. They are struggling to provide care to a steadily increasing number of unscheduled visits [1]. In recent years, EDs are suffering from increasing stress due to a remarkable growth in demand, limited productivity, and reduced budgets which mostly lead to overcrowding in EDs [2]. As a consequence, patient congestion and long waiting times in EDs are one of the most common problems in public hospitals [3]. Moreover, patients expect that services are well organized from a *customer* perspective. That is, the service management has changed from optimizing resources usage to finding the tradeoff between quality of service for patients and operational efficiency for healthcare providers [4]. For this purpose, ED managers must control problems related to process flow (patients and information), as well as internal restructuring reflected by resource pooling [5]. However, EDs are highly complex environments: patient arrival rates vary over time, patients' care paths depend on urgency and pathology, resources may not be suitable for treating all types of patient,

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urgent patients get priority over non-urgent patients, admitted patients often keep occupying ED because of the bed unavailability in hospital [6]. The efficient management of patient flow in EDs has thus become an urgent issue for many hospital administrations. The study of complex systems poses unique challenges: some of our most powerful mathematical tools, particularly methods involving fixed points and attractors are of limited help in understanding the development of complex systems [7].

From a computation theoretical perspective, simulation of a system can be defined as an "*imitation (on a computer) of a system as it progresses through time*" [8]. Research in modeling and simulation are mostly derived from system dynamics [9] and discrete event simulation [10–12] methods. Both methods focus on building models to directly mimic the system-level behavior but they differ in how the system is modeled and how time is evolved in simulation [13]. In contrast to system dynamics and discrete event simulation methodologies, an agent-based model (ABM) focuses on modeling individuals, interactions between individuals, and interactions with a physical or influential surrounding environment [14]. The ABM is a class of computational models for simulating actions and interactions of agents (individual or collective entities such as a medical imaging test-room) with a view to assess their effects on the system as a whole. Each agent is guided by a set of programmed rules with proper transition probability, and is capable of acting independently. ABM is commonly used to study complex systems since it shows behavior more like our idea of how the real system works, and such models can be constructed even if we do not understand the systematic behavior. That is, we only need to know how a system component (modeled as an individual agent) behaves in the system and how it interacts with others in order to set the model to an initial configuration and watch it evolve over time [15]. Since the emergent behavior of a complex system depends upon the individual-level behavior of its components, these agent models can be used to gain insight into the emergent behavior of complex systems [16].

Furthermore, the system analysis in many application domains is not only about accuracy in prediction, as interpretability is also extremely important to have transparency in predictive modeling. Because domain experts prefer models that emulate the way a human expert makes decisions over "black box" predictive models. They want to understand how the prediction is made, and how the predicted scenario is caused by the individual-level behavior [17]. Consequently, the ABM is capable of answering questions such as why the system behaves the way it does.

1.2. Literature review

Healthcare simulation is used for dynamic as opposed to static analysis [18]. Today, many researchers are interested in modeling and simulating the operation of EDs because it can help managers carry out different kinds of analyses, such as: (1) resource utilization (human, equipment, and space) for alternative scheduling and allocation policies [19, 20], (2) finding the most influential factors affecting the performance of the system in a given situation [21], (3) exploring the interrelationships between individual-level behavior and system-level performance under various scenarios [22] and, (4) estimating system robustness under unexpected situations (e.g. the outbreak of infectious diseases) [20, 23, 24]. An important feature of simulation modeling is that it allows us to evaluate various scenarios so that "what-if" analyses can be performed, and improvement initiatives can be taken [25]. Once developed and properly validated, the model can be used to predict, explain, and optimize the system performance without the commitment of any physical resources or interruption to the system. Gul and Guneri [26] made a comprehensive review of ED simulation studies, in which they stated that ABM has a significant capability and is becoming an emerging methodology for ED simulation applications. They also found that papers from 2011 to 2015 with ABM, and ABM and other simulation techniques, are steadily increasing compared to before 2011.

Regarding the model-based optimization of EDs, Oh and Novotny et al. [27] proposed a simulation-based decision support tool to improve emergency department throughput. Authors select a throughput time goal of arrival to departure under 3h for 80% of ED patients, then they use a simulation-based decision support framework to achieve this performance goal. At the end, 30% improvement in average patient length of stay was achieved after their simulation-supported redesign. Regarding patient discharge strategy optimization (discussed in §2.2), Nezamoddini and Khasawneh [28] used a model-based method to optimize the strategy of transferring patients between hospitals in order to minimize patient wait times without increasing the number of required resources.

1.3. Contribution and objective

In this article, a simulation model for EDs is developed to enhance the understanding of an ED's complexity, completely evaluate "what-if" scenarios (analysis) and perform experiments with the simulator prior to making changes to an ED system (prediction). The ED managers can use it to quantify the impact of proposals on patient flow as well as system efficiency prior to implementation. This research was devised in close collaboration with experienced ED staff in the Hospital Taulì de Sabadell (a University tertiary level hospital in Barcelona, Spain that provides care service to a catchment area of 500,000 people, and attends more than 160,000 patients per year in the ED). With good flexibility and scalability the simulator provides, it has been used to study the Methicillin-resistant Staphylococcus aureus (MRSA) transmission in EDs [29, 30], and as a sensor of EDs to provide data for knowledge discovery [31]. Some case studies and demo applications carried out by using this simulator have been previously published in conference proceedings [32, 33].

The ultimate objective of this work is to describe an accurate model of EDs. This model can be used to predict behavior of EDs under various conditions such as staffing change, patient scheduling, physical resources resize and influx of patients (e.g. during the influenza season). The presented work is from a long-term project which aims to develop a generic ABM of EDs:

1. In the management of EDs, the simulator can work as a part of the decision support system to quantify the cost and benefits of proposals (e.g. [32]).
2. Making the ED simulator work as a platform to study ED related problems. For example, researchers use this simulator to study MRSA transmission in EDs [29, 30].
3. The framework developed in simulating EDs can be used to build a full model of the integrated care system [34]. Then the final model will represent a comprehensive tool to quantitatively evaluate prospective planned changes to the integrated care system (e.g. [35]).

So, the main focus of this paper is to systematically detail how the ED simulator was built. Regarding demo applications that show the capability of the simulator, we refer readers to [29–33, 35]. For example, [32] presents the application of this ED simulator for solving an overcrowding problem caused by a flu outbreak. By using an ED simulator, authors [32] also studied the influence of ambulance response time on ED overcrowding. Some interesting case studies are presented in [33] to show the advantage of using this agent-based simulator towards explaining the complex behavior of an ED.

1.4. Organization of the paper

The rest of the paper is structured as follows: §2 describes the conceptual model of ED observed from real operation data and the involvement of experienced ED staff. §3 gives a patient arrival model. The full ABM of ED is detailed in §4, which contains two parts, §4.1 describes the model of agents, and §4.2 describes the interaction model among agents as well as agents with the environment. The §5 formulates the agent models into algorithms and implemented with a programming language. Once the computational model has been achieved, §6 presents methods to design experiments as well as execute the model for predicting, explanation and optimization. The simulation results that validate the model are presented in §7. Finally, §8 closes the article with our conclusions and potential future contributions.

2. Conceptual model of emergency departments

Typical EDs have common interacting elements such as doctors (physicians), nurses, technicians, receptionists, beds, medical devices that are interconnected via flows of information and processes (registration, triage, diagnostic, discharge). All elements methodically interact with each other to produce diagnoses, treatments, and information. The ED studied in this research is focused on the Spanish public hospital based EDs. However, we are confident that the proposed method and the simulation framework can be used for other EDs.

This section gives a conceptual model of generic EDs. This conceptual model was carried out by analyzing 4-year's historical data provided by a typical ED, and conducting interviews with experienced ED staff. The analysis of historical operation data determines the nature of statistical distributions followed by each process and their corresponding parameters, and the state transition probabilities. Meanwhile, the participation of experienced ED staff helps us to establish a comprehensive understanding of the hospital EDs and focus on considering significant features of ED in modeling and simulation.

2.1. Process in ED

Typically, a patient, which is modeled as an agent in this study, enters an ED through one of two ways: by themselves or by ambulance. As shown in Figure 1, patients who arrive by themselves need to walk to the registration window and briefly give their personal information to the registration staff. After that, they have to stay in a first-come, first-served (FCFS) waiting room. Then a patient goes to the triage box and interact with a triage nurse (modeled as an agent) once get notified by the information system. Triage consists of a brief assessment of the patient's physical condition and an acuity level (AL) will be assigned to the patient according to their severity. After triage, patients will wait in another waiting room before entering the diagnosis and treatment area. The patients who arrive by ambulance are registered and triaged in the ambulance and can go to the second waiting room directly. The Spanish scale of triage is very similar to the worldwide Canadian Emergency Department Triage and Acuity Scale [36, 37]. The scale comprises 5 levels, with level 1 representing the most critical (resuscitation), and level 5 being the least critical (non-urgent). The patient's AL also determines the place where they will be treated and the order and priority with which the patient must be attended. The registration and triage service are in FCFS manner for all the patients, whereas entering the diagnosis and treatment area is acuity-level-dependent FCFS (e.g. patients with AL 1 have the highest priority).

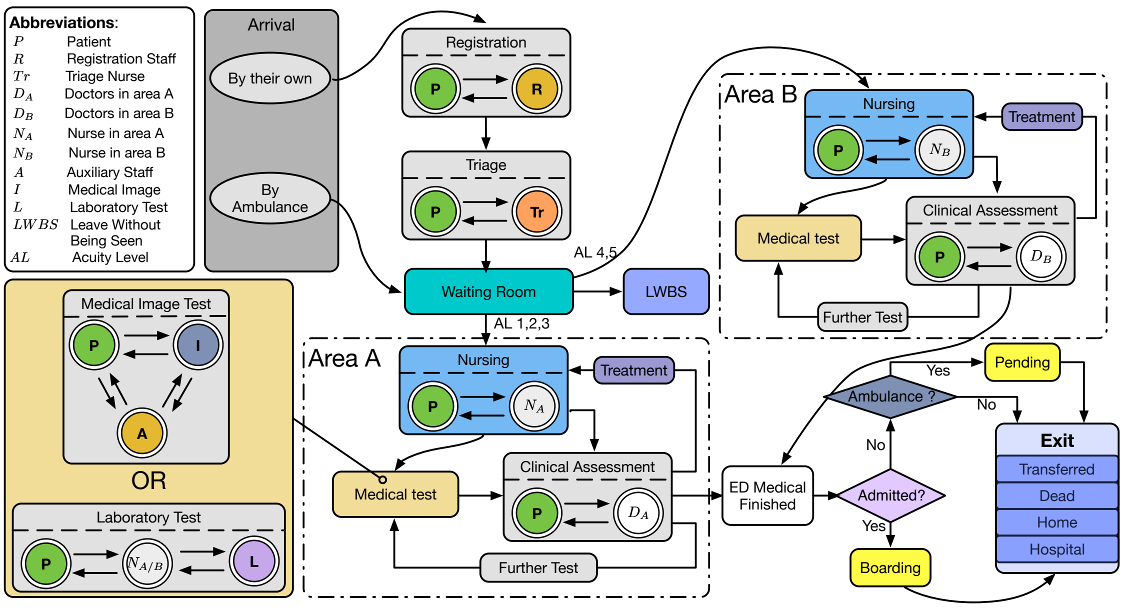


Figure 1: The emergency department operation process as well as interactions among its components. The group of two parallel lines with arrow stands for interaction. Patient flow is guided by the emergency department information system.

In most Spanish EDs, there are two treatment areas (labeled as area A and B in this study, see Figure 1) which operate independently to provide the diagnosis and treatment service. Area A is for patients with AL 1, 2, and 3 and consists of small rooms containing essential medical equipment and supplies that can be used for patient's treatment, called careboxes. Area A patients tend to stay in their carebox. When a doctor assigns a medical image test which requires moving the patient, they are assisted by auxiliary staff. Area B, also called fast track, is a dedicated stream of resources to process patients of AL 4 and 5 more quickly. It consists of attention boxes in which doctors and nurses interact with patients, and a large waiting room in which all patients will remain while not having interaction.

The patient flow as well as interactions (the group of two parallel lines) in general EDs are shown in Figure 1. It is worth noting that the process in Figure 1 is a generic routing for all patients. It can be seen as a multi-class queuing system with probabilistic routing. Every patient who comes through the door is an unknown, with a condition that unfolds over time in a functionally non-deterministic way. Theoretically speaking, no two paths through this "system" are the same for any two patients. Accordingly, from a patient's perspective, they are either receiving service (interacting) or waiting for resources (healthcare staff or physical resources like beds, testing equipment) becoming available. Each patient, healthcare staff, carebox and test room (medical image and laboratory) will be modeled as an agent in §4.

2.2. Discharge

As shown in Figure 1, there are four possible destinations when patients finish their treatment: admitted to hospital, go home, transferred to another hospital or dead. If a patient is admitted to hospital and there is no bed available in the hospital, they have to stay in the ED. As a consequence, hospital bed availability will strongly affect patients' length of stay (LoS) in ED and throughput (the number of attended patients per day) of the ED. Results in [38] show that the increased hospital occupancy has strong association with LoS in ED for admitted patients. Because the pending patients in ED prevents EDs from serving new patients in a timely manner and results in longer LoS as well as a percentage of patients who left without being seen. Therefore, although our work was focused on modeling EDs, the model of bed availability in the hospital should be carefully considered as well. Based on the actual historical boarding time, a Poisson distribution is used to fit the bed availability pattern in hospital.

Similarly, for those patients who will go home or be transferred to another hospital, some of them will request the ambulance service. Since the ambulance service is provided by a service center, it is common that there is a delay between requesting and when it becomes available. The same as bed occupancy in hospital, the response time of the ambulance service also affects the ED's behavior because patients keep using physical resources of the ED during their wait. Having said that, it is also necessary to include the model of ambulance response time as part of the ED model to analyze the degree of the impact on an ED. Based upon 4-year's real data, a Gamma distribution $X \sim \Gamma(k, \theta)$ was used to fit the length of ambulance response time. A detailed ambulance response time model can be found in Ref. [32]. Moreover, patient discharge is often delayed because staff are tied up with more urgent patients [39]. This indicates that staffing and scheduling also have a widespread effect on all areas of the ED.

In summary, patients that have been discharged from the ED either leave immediately without requesting the ambulance service or undergo other waiting phases: boarding to hospital or waiting for the ambulance service. While patients remain blocked or boarded in an ED bed, they prevent other patients from starting treatment, which might lead to ED overcrowding [6].

2.3. Door-to-doctor time and leave without being seen

The length of waiting before seeing a doctor is known as door-to-doctor time. According to the public healthcare regulations in Spain [36], for those patients triaged as AL-1, 98% of them should be attended immediately, 85% of the patients with AL-2 must be attended immediately by a nurse and within 7 minutes by a doctor, and for AL-3, 80%, within 15 minutes; AL-4, 75%, within 30 minutes; AL-5, 70%, within 40 minutes [36]. If there is no free space, upcoming patients are treated in the corridor (considered as a virtual carebox in this model). However, the service capability of the healthcare staff is limited, some patients, especially those triaged as low AL may face long waiting time and they may leave without being seen (LWBS). As investigated by Ding et al. [40], it is common to see above 6% of patients LWBS due to physician unavailability.

LWBS is a crucial efficiency and effectiveness metric for public EDs. Although a patient's decision to LWBS is influenced by many factors [41, 42], we assume this decision only depends on the patient's waiting time. Based on questionnaires and on-site interviews, we use an acuity-level-dependent Triangular distribution ($X \sim Triangular(a, b, c)$) to fit the length of time that patients stated they were willing to wait before LWBS, the parameters a, b and c are related with patient's age, AL and gender. Simulation users also can define a sub-model for patient's tolerance of waiting.

3. Patient arrival model

Patient arrival is the input of an ED, which has direct influence on the system behavior. As the simulator is not only designed to imitate an actual situation but also for studying system response under unexperienced scenarios. A precise and customizable patient arrival model is crucial to representing a given scenario. Specifically, the patient arrival model should reflect the general pattern of patient arrival, and it should be easy for users to customize for a given scenario. To an ED, the patient's arrival pattern is characterized by arrival *rate* (e.g. number of patients in each hour of a day) and patients' *key characteristics* (e.g. age and severity). In this section, we build an arrival model based on the synthesis of various opinions from expert ED staff, on-site observation, and 4-year's historical data from the Hospital Taulì de Sabadell.

3.1. Rate

We found that the arrival rate fluctuated significantly through the day, and the number of daily arrival patients was influenced by day of week and season. For instance, EDs get fewer patients in August, patient arrival reaches a maximum on Monday and minimum on Saturday. Consequently, we modeled the patient arrival rate in a time interval of one week (by using another dataset, [43] also find that it is important to take a weekly view rather than the common daily view). The arrival rate model includes a table of normalized (proportion of weekly arrival) hourly arrival rate $R_{ar}[hour, day]$ in one week and the number of weekly patient arrivals $N_{ar}[week]$ ($N_{ar}[week]$ can be used to reflect the seasonal effect). Accordingly, for a given *hour* in *day* and *week*, the number of patient arrivals in a specific hour (R_{hour}) can be computed with $R_{hour} = N_{ar}[week] \times R_{ar}[hour, day]$. Subsequently, the arrival process in one hour was fitted by a non-homogenous Poisson process [44]. The probability density function of patient inter-arrival time can be expressed as follows.

$$f\,(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (k \geq 0) \tag{1}$$

$$\lambda = \frac{60}{N_{ar}[week] \times R_{ar}[hour, day]} \tag{2}$$

Where, e is Euler's number, $k!$ is the factorial of k, k denotes the inter-arrival time between the nth patient and the $(n+1)$th, and λ denotes the inverse of average arrival rate (number of patients per minute). So, in implemention, we generate a random number k_1 from the Poisson distribution $X \sim Poi(\lambda)$ with parameter λ calculated by Equation 2 in the first minute of each hour $h\,(0 \leq h \leq 23)$. The value of k_1 means that the first patient will arrive at time $h : k_1 : 00$. Subsequently, when simulation time is up to $h : k_1 : 00$, a new patient (object) will be created and send to a registration waiting room. At the same time, we generate another random number k_2 which represents that the second patient will arrive at time $h : (k_1 + k_2) : 00$. In this way, the nth patient will arrive at time $h : (\sum_{i=1}^{n} k_i) : 00\,(0 \leq \sum_{i=1}^{n} k_i \leq 59)$. The flow of the patient arrival model is shown in Algorithm 1, this procedure will be called at every simulation time-step (one time-step represents a period of actual flow time, it is 30 seconds by default in our implementation).

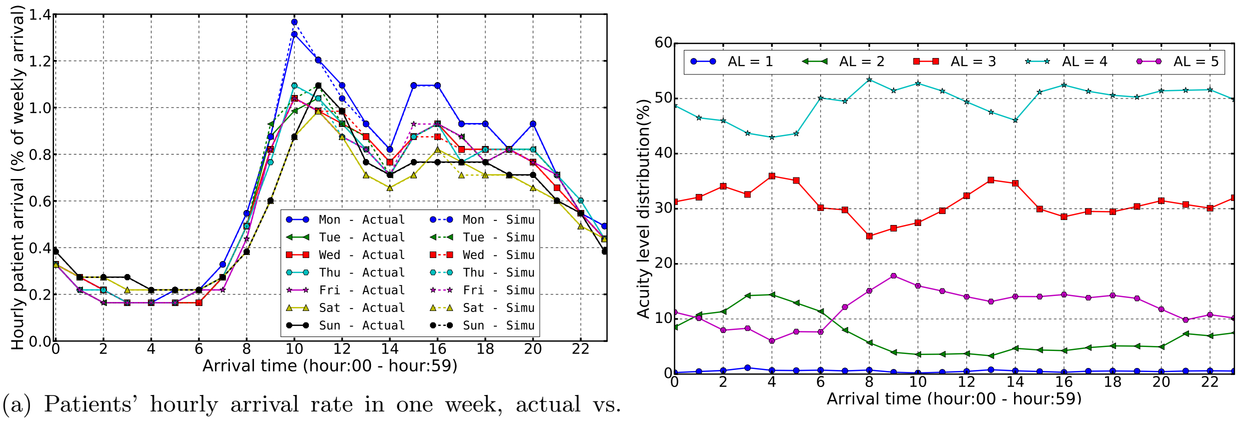
With the above-descried patient arrival model and real operation data of 2014, Figure 2a shows 12-month simulated results compared with actual data (broken versus solid lines). It is clear that the simulation result is very close to actual data at an hourly rate. The weekly arrival (the sum of hourly arrival), simulation: 1829 versus actual: 1826 is very close as well.

3.2. Key characteristic

We considered arrival patients' severity (measured by AL) and age as their key characteristics. Observing from actual data, we found that the severity of arrival patients also fluctuates remarkably through the day (Figure 2b) but no significant effects were found in longer time periods. The data illustrated in Figure 2b was retrieved from 12-month actual data from 2014. In our model, a Gibbs sampler (a Markov chain Monte Carlo algorithm) [45] was used to obtain a sequence of values to represent patients' severity in simulation. With enough samples, these values will be able to approximate the distribution data shown in Figure 2b. Similarly, the age distribution was extracted from actual data and then fitted with a Gibbs sampler from

Algorithm 1 Patient arrival model algorithm

procedure PATIENTARRIVAL
    static $k \leftarrow 0$ ▷ define k as a static variable
    if $now.minute == 0$ **and** $now.second == 0$ **then**
        $\lambda \leftarrow$ **call** $computeLambda(datetime.now())$ ▷ implementation of Equation (2)
        $k \leftarrow random.poisson(\lambda)$ ▷ arrival time of the first patient in this hour
    end if
    if $now.minute == k$ **and** $now.second == 0$ **then** ▷ arrive at time hour:k:00
        call $createPatient()$ ▷ create patient; characteristics will be set
        $k \leftarrow k + random.poisson(\lambda)$ ▷ arrival time of the next patient
    end if
end procedure



(a) Patients' hourly arrival rate in one week, actual vs. simulation.

(b) Arrival patients' acuity level distribution (hourly).

Figure 2: Patient arrival model: hourly arrival rate (quantified by percentage of weekly arrival rate), simulation vs. actual (average in 12 months), and arrival patients' acuity level distribution (extracted from 12-month actual data, 2014).

the distribution. That is, the model (a Gibbs sampler for patients' severity and age) describes the pattern of patient arrival, the parameters for characterizing actual behavior depend upon the given simulation scenarios, and simulation users can customize their scenarios through modifying the corresponding parameters.

4. Agent-based model of emergency departments

The conceptual model described in §2 provides a brief understanding of the complex operations in EDs. It is clear that care services in ED are carried out by interactions between patient and components of ED (human and material resources). Since quantified performance measurement and evaluation of a system are more important for continual improvements in the system, the conceptual model must be formulated into algorithms and implemented by programming languages. Then the model can be executed to quantify predictions, explain phenomena and optimize performance on a systematical level. Respectively, we first describe agent models that represent the ED system components in §4.1. Then in §4.2, we describe the interaction models that connect agents to represent the service in ED.

4.1. Design of agent models

A way to precisely define the agents and their interactions must be provided in order to study complex systems with ABM techniques [7]. The actions of agents usually depend on the signals they receive. Thus, the agents can be formulated as an IF/THEN structure: IF [signal vector x is present] THEN [execute act y] [7]. If an agent is busy with an interaction while a signal is being presented, the presented signal will be pushed into its task queue. This behavioral model structure is: (1) easy to abstract from agents' actual behavior in the real system, (2) in accord with KISS principle (keep it simple, stupid), and (3) easy to be converted into a programming language. The following sub-sections 4.1.1 - 4.1.8 will detail behavioral rules of all agents in an ED with the IF/THEN structure. For convenience, we define notations in Table 1 to represent a specific group of agents.

Table 1: Notations used in this model.

Notations	**Description**
i	ID of agent, it is a unique ID among all the agents in the model.
N_i^{CB}	a set of careboxes under the responsibility of nurse i.
N_i^P	a set of patients (mainly for patients in area B) under the responsibility of nurse i.
D_i^P	a set of patients under the responsibility of doctor i.
IS	The information system in the ED, a system for communicating and coordinating among staff, patient and test-room, also treated as an agent in this model.

4.1.1. Carebox

As described in §2.1, area A is made up of careboxes that containing essential medical equipment and supplies. In this model, we consider a carebox as an agent since it can be reserved. A carebox agent does not have behavior rule and will be either idle or occupied.

4.1.2. Patients

Patients in both areas A and B are guided by the IS, i.e. they go to the corresponding place when they get notified. During all the process in the ED, the patient alternates between two states: receiving treatment or waiting (i.e. waiting for a doctor, nurse, medical testing service/result). The patients in area A are solely guided by service providers, i.e. doctors, nurses and auxiliaries. They stay in their carebox when there is no interaction with service providers. Therefore, patients' LoS in the ED is the sum of all activities (meeting with doctor, medical tests, and having a rest to wait for drug therapies to take effect) they have to attend to, and time spent on waiting for resources (including test rooms, doctors, nurses and auxiliaries) to become available. The behavioral rules of patients are given in Table 2. It is worth noting that the time spent on waiting until drug therapies take effect (t_{drug}) is significant because it is usually the longest part of LoS. In this study, we fitted the t_{drug} with AL dependent statistical distributions. The parameters of the distributions depend upon the patient's AL and are calibrated in the model tuning process.

Table 2: Behavioral rules of patients.

IF	THEN
notified by *IS* (before entering treatment area).	go to the corresponding place in the notification.
no requests from *IS* (before entering treatment area).	stay in waiting room.
no interaction requested by healthcare staff (nurse, doctor or auxiliary).	remain in carebox (for patients in area A).
no requests from *IS* or healthcare staff.	stay in waiting room (for patients in area B).
notified by *IS* (in area B).	go to diagnosis room or medical image test-room as indicated in the notification.
needs additional help.	ask nurse through *IS*.

4.1.3. Registration staff and triage nurse

The behavior of triage nurses and registration staff is similar, so they are described together in Table 3. The service time depends on the experience (junior or senior) of registration staff and triage nurses, not on patient characteristics.

Table 3: Behavioral rules of registration staff / triage nurse.

IF	THEN
time to work (start shift).	interact with colleague in previous shift, take over materials from them.
no patient in front of the desk/window.	keep waiting for patient (IDLE).
one patient is waiting in front of the desk.	interact with patient for registration/triage.
shifting of duty time is up (end shift).	accomplish work at hand, interact with colleague in following shift, hand over requested material.

4.1.4. Doctor

Doctors in area A have to walk around the area in order to interact with their patients in careboxes because patients in area A are not allowed to move by themselves. The time it takes for the doctor to move is important to consider as it is not constant and significant for the system efficiency. Doctors in area B sit in their office waiting for patient to come. Considering that the waiting room is not far from the doctor's room in area B, the time it takes for the patient to move is constant and negligible. Detailed behavioral rules of doctors are shown in Table 4.

The time for each interaction (service time) depends on many factors. According to the real behavioral data and findings in queue theory [46], the duration of service time for a specific worker can be fitted with exponential distribution ($Exp(\lambda_{st})$). The parameter λ_{st} is AL (al), service type (s) and service provider's experience (sp) dependent in this model (see Equation 3).

$$T_{st} = Exp(\lambda_{st}) + t_{move}, \quad \lambda_{st} = \gamma_n \cdot f(s, sp, al) \tag{3}$$

Where, t_{move} is the time taken on movement, which depends on the location of the patient's carebox. t_{move} is set as zero in area B. On-site interviews and staff experience show that the first meeting with a patient takes longer than follow-ups and most patients (especially in area A) need several meetings with their doctor. Therefore, we added a proportionality coefficient parameter (γ_n) for the service time in Equation 3,

Table 4: Behavioral rules of doctors, rules specified with *in area A* means doctors in area A, otherwise applies to area A and B.

IF	THEN
time to work.	interact with doctor in previous shift, take over patients from them.
no task assigned by *IS* (task queue is empty).	stay in their office (IDLE).
IS notifies a new patients in carebox i (in area A) / A new patient comes into office (in area B).	move to carebox i (in area A), perform first-interaction, make treatment plan.
IS notifies: the test report for one of the patients in set D_i^P is ready to review.	review medical test report, walk to the carebox (in area A) if necessary, and make follow-up treatment plan (do more test, drug therapy, discharge or admit to hospital).
scheduled drug therapy time of any patient in set D_i^P is up.	walk to the carebox (in area A), check effect of drug therapy, and make follow-up treatment plan.
shifting of duty time is up.	accomplish work at hand, interact with doctors in following shift, hand over all the patients in D_i^P.

i.e. the same statistical model, but a different scale. The γ_n thus represents the proportionality coefficient for the first meeting with patient and follow-ups (e.g. 1.0 for first meeting and 0.7 for follow-ups). The function is identified and calibrated with real operation records of the target ED. Note that the service model described by Equation 3 is also applied for nurses but with different parameter values. While service time depends on the service provider and patient characteristics, routing probabilities only depend on patient characteristics.

4.1.5. Nurse

Similar to doctors, the nurses in area A have to move to a patient's carebox to provide service and t_{move} is crucial to consider. We use the same model of Equation 3 for the nurses' service time but with different parameters. The IF/THEN behavioral rules of nurses are detailed in Table 5.

Table 5: Behavioral rules of nurses, rules specified with *in area A* means nurses in area A, otherwise applies to area A and B.

IF	THEN
time to work.	interact with nurse in previous shift, take over patients from them.
no task assigned by *IS* (task queue is empty).	stay in the nurse room.
doctor assigned laboratory test to one of the patients in set N_i^P.	walk to the patient (to carebox N_i^{CB} in area A), take sample from patient.
drug therapy assigned to one of the patients in set N_i^P by a doctor.	go to the pharmacy, take pill and then walk to the patient's place for treatment.
IS notifies an additional-help call from patient in set N_i^P.	go to the patient (to carebox N_i^{CB} in area A).
Periodic checking time is up.	Check every patient's physical condition in set N_i^P.
doctor discharged one patient in set N_i^P.	help patient leaving ED.
shifting of duty time is up.	accomplish task at hand, interact with nurses in following shift, hand over all the patients in set N_i^P.

It can be seen from Table 5 that all the behaviors are driven by the *IS*. Since we assume that nurses always behave according to the rules, the uncertainties of nurses' behavior are due to the uncertainties of patients.

4.1.6. Auxiliary technician

Auxiliary technicians work in area A to assist patients to move around for medical testing. The behavior of auxiliary technicians is simple but crucial to consider because their behavior also has a significant impact on system performance. For example, a shortage of technicians will result in delays in patients' tests, and further affects the efficiency of test-rooms. This chain reaction will finally affect the throughput of the ED system. Different from doctors and nurses, all the technicians share one task queue, i.e. when there is a request and if there are idle technicians, one of them will take the task, otherwise, the task will be pushed into the technician group's task queue. Therefore, they have only one IF/THEN behavioral rule, i.e. one of the available technicians goes to perform the task when notified by the IS.

4.1.7. Medical image test-room

Medical imaging service is the process of creating visual representations of the interior of a body for clinical analysis and medical intervention. Although the medical image test-room is comprised of equipment and technicians, we model this unit as a single agent to ignore unnecessary complexity. This agent will interact with patients and auxiliary technicians. There are several kinds of medical imaging services, such as CT-scan, B-scan ultrasonography, X-Ray and MRI-scan which differ in service time. The type of test is determined by a doctor based on patient's characteristics (AL, previous test). Different test-rooms perform independently to provide service simultaneously. The service time was fitted with exponential distributions based upon real data analysis and findings in queue theory [46]. The IF/THEN behavioral rule of medical imaging test-room is described in Table 6.

Table 6: Behavioral rules of medical image test-room.

IF	**THEN**
no patient waiting outside.	waiting for patient (IDLE).
patient with auxiliary staff waiting outside, and test-room is ready.	interact with patient and accompanied auxiliary staff.
physical test finished.	process test results, and send to the corresponding doctor through IS.

4.1.8. Laboratory test-room

The laboratory test-rooms receive patients' samples (e.g. blood) taken by a nurse, and the machine analyzes them one by one in FCFS manner. Multiple machines work independently to analyze multiple samples simultaneously. Each machine requires maintenance service every 24 hours. The maintenance service takes up to one hour and during which samples cannot be processed. Here, each machines is considered as a whole as an agent. The process time of different kinds of analyses are separately fitted with exponential distribution. The parameters of the distribution are based on the specification of machines and carefully calibrated based upon real data. The behavioral rules are detailed in Table 7.

Table 7: Behavioral rules of laboratory test-room.

IF	**THEN**
no sample in the queue.	waiting for sample (IDLE).
new sample(s) waiting in the queue, and there are free analyzing machine(s).	detach sample(s) to free machine(s).
machine(s) completed the analysis.	catch results and send to the corresponding doctor through IS.
daily machine maintenance time is up.	start maintaining when machine completes current task.

4.2. Interaction Model

Agent models are just one aspect of an agent-based modeling and simulation system. The interaction model which connects all agents to form a dynamic system is another aspect. Since hospitals have strict behavioral rules, it is reasonable to assume that all agents in EDs behave according to their behavior rules. Figure 1 shows that there are one-to-one interactions (e.g. doctor with patient), one-to-n interactions (like *IS* with patients), and triangular interactions (e.g. test-room, patient and auxiliary staff).

Agent models described in §4.1 are defined from a single agent's perspective. To accurately represent a "live" agent in simulation, besides behavioral rules, each agent has its own state variables for determining their current state. Both the IF/THEN behavioral rules and the state variable determine the action. For example, if a patient's location state variable indicates that he/she stays in the first waiting room (i.e. *location = the 1st waiting room*), it is sure that he/she is waiting for the triage service instead of others. Table II in our previous paper [47] gives another example on representing agents' state by their variables. Therefore, a combination of these state variable values are capable of representing a huge number of agent states and states are easily added/removed by adding/removing elements in the set of state variables and their corresponding behavior. For the study of other ED related problems, for example, the study of MRSA propagation in the ED [29, 30], some new state variables and their possible values are added to gain insights into how the parameters evolve over time (e.g. routes of bacteria transmission). With the same approach, further functionality of the research object will emerge from these new states.

5. Model Implementation and data collection

5.1. Model implementation

The nonlinearities and interactions among agents over time and space can lead to such complexity that it is only possible to understand through simulation [48]. The full model has been implemented in NetLogo [49] simulation environment, which is an agent-based programming language and integrated modeling environment. We use the "turtles" in NetLogo environment to represent agents described in §4.1. During simulation each agent returns to the inactive state and check its task list after completing an interaction. The next task will be chosen from the list according to the priority policy. Thus, agent models described in §4.1 can be easily translated into the NetLogo programming language with a state machine structure. Since the systemic key performance indicators (KPIs) are extracted from detailed interaction information among all the agents, a proper way to collect this atomic interacting data should be carefully designed.

5.2. Atomic data collecting

An ABM of an ED is comprised of agents that represent system components as well as interactions between agents. Thus the raw simulation results are atomic data about agents' state transition over time, and interaction between agents. This atomic data is the source of knowledge for a better understanding of the complex system. Therefore, simulation output from the agent-based simulator is subjected to extract the system-level behavior information. However, one reason why emergent behavior of a complex system is hard to predict is that the number of interactions between components increases with the number of components. The interaction data and the state information of the system are thus massive. In the data collection process of a real system, we tend to collect as much data as possible to cover the behavior of the real system. Whereas in simulation, data monitoring should be focused on the goal of analyzing because the simulation process is reproducible (using the same model configuration and random seed). To this end, we designed a customizable data monitoring layer between the individual-level behavior simulator and the data processing layer. More specifically, like in a real situation, we use sensors to detect events and changes in the environment. In this simulation model, we implemented a virtual sensor that is attached to each agent (host). The "sensor" is capable of collecting events of its host agent, and can be enabled/disabled by the simulation user. Thus this data monitoring layer is a set of "sensors" which enable simulation users to customize their data collecting according to their analysis requirements without accessing the source code. Moreover, some "sensors" also provide simple data processing methods to carry out basic analysis (e.g.

maximum, minimum, average, median value, standard deviation) in order to reduce the size of simulation output without affecting the final analysis (e.g. in cross-scenario analysis cases).

This customizable data monitoring layer has been implemented in two parts: an independent application with graphical user interface which enables simulation users to customize data-collecting behavior, and a data-collection program along with the simulator to record and write data to comma-separated values (.csv) files. These two parts communicate through a configuration file before starting simulation.

Considering the nature of an agent-based simulator, we classified the raw simulation data into two categories: environment state and interaction information. The state information, e.g. test-room occupancy and number of patients in waiting room, are sampled in a given time interval (sample frequency, e.g. output the length of a test-room queue every 5 minutes for indicating its occupancy). The interaction information contains records of all the interaction among agents. This interaction information is structured as five Ws (Who, What, When, Where, Why) and one H (How long it takes).

6. Experiment design and model execution

6.1. Scenario design

The term "scenario" in this article represents a set of parameter values which characterizes an ED. In our simulator, only the model (agent model and interaction model) was implemented in NetLogo, the value of parameters which characterize the model are loaded from configuration files. Therefore, simulation users can design their experiments in a form of configuration files without accessing the source code. For example in a decision support process, users can analyze their requirement (e.g. overcrowding, influx of patients) at system-level, then design experiments in individual-level (i.e., parameters to characterize agents' model). Finally the system-level systemic KPIs predicted by simulation will be extracted from individual-level interaction data. The state of the ED is then assessed by using selected KPIs. If the ED reaches or is in a degraded or critical state, corrective solutions are proposed, and then new simulations will be performed to verify the effectiveness of these solutions. This process is repeated until the ED returns to a normal state as expected.

6.2. Model execution in cluster

Agent-based simulation is computationally expensive. The ability to test a large number of scenarios for "what-if" analysis in a short time period makes simulation a widespread tool in decision supporting and operation research. This sub-section describes a framework to efficiently execute the model that was implemented in the NetLogo environment. Due to the inherent nature of patient flow with characteristics such as stochastic arrivals, stochastic service times, and uncertainties in patient routings, the agent-based ED model has stochastic characteristics by nature. Consequently, results from a single execution may not be statistically reliable, so repeating one scenario with different random seeds becomes necessary and challenging (see §6.4). High performance computing techniques are prominent means of leveraging the complex simulation power of ABMs. Since there is no data dependency among different scenarios or repetitions of the same scenario, the master-worker execution model becomes a good choice. However, as execution time of different scenarios or even repetitions of the same scenario are different, load balance should be carefully considered. Given this, we designed an execution engine to launch and execute the ABM on cluster with NetLogo. A dynamic task assignment over the master-worker mechanism was used to achieve load balance. The execution model is illustrated in Figure 3, in which MPI (Message Passing Interface) is used to manage and communicate among master and worker processes.

As the meaningful information (systemic behavior indicators) is extracted from interaction records, and the size of interaction data increases with the number of agents, it is better to process data in the same computing node that simulates the scenario to avoid large data movement between master and workers. More specifically in this framework, the number of workers is equal to the total number of CPU cores, and the master will be assigned to one of the nodes (Figure 3). When one scenario finishes its execution, the data analysis program will be launched to analyze the simulation data locally on the same computing node. Then the processed system-level KPIs will be sent back to the master. The NetLogo controlling API

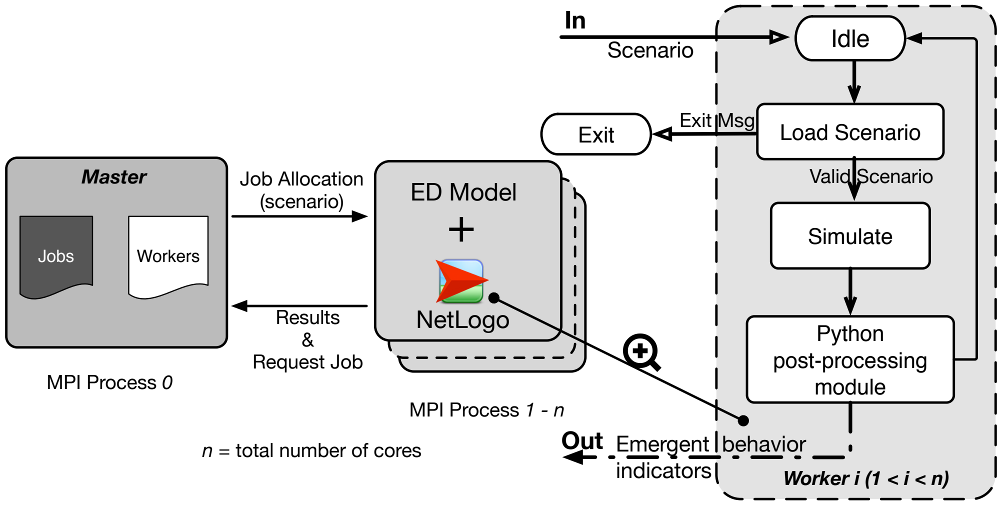


Figure 3: The master-worker execution framework for ABMs on a cluster. Atomic data will be analyzed natively in the same node, and only systemic information will be sent back to master.

(which comes with NetLogo.jar from the released version) was used to invoke and control NetLogo. That is, when a new execution is assigned by the master, instead of reloading the NetLogo environment (which takes around 30 seconds), the controlling API will initialize the model as well as the NetLogo environment with parameters in the scenario. Thus, NetLogo will only need to be launched once for each worker. The master will send an "exit message" to a worker if there is no further task for that worker, and the worker will release the NetLogo instance and exit. Specifically, MPI library and C programming language were used to distribute and balance simulation task, Java was used to take full control of NetLogo (via controlling API), and Python was used for simulation data analysis. Furthermore, this model execution framework is one way of addressing the computational complexity of such systems, which is also useful for model parameter calibration and simulation based optimization. Although the presented framework was designed to speed up our simulation study, we are confident that it also has the potential of being used by for other ABMs that are implemented with NetLogo.

6.3. Transient phase simulation

The ED simulation model is a typical steady-state system that neither has starting points nor requires stopping during the simulation process [50]. We thus use a period of warm-up time as the transient period before simulation in order to remove the initial bias. To determine the proper length for warming up, we first used the linear regression approach [51] to identify the end of the transient state. This approach uses the least-squares method to determine if the linear regression slope coefficient is close enough to zero for a specific range of observations, i.e. the transient state finishes when the slope is close to zero. Then based upon the transient period length (normally less than 100 hours), the warm up period has been set at a large margin of one week (168 hours). The data collecting layer will start to record all the requested atomic data (i.e. enable "sensors" according to user's configuration) after warm-up.

6.4. Replication

In this ED model, agents' behavior is fitted with statistical distribution and most of the KPIs (e.g. LoS, door-to-doctor time) are statistical indicators. Therefore, according to the law of large numbers [52], a minimum sample size should be guaranteed in order to make KPIs statistically reliable.

In probability theory, Chebyshev's inequality guarantees that in any probability distribution, "nearly all" values are close to the mean [44] . Specifically, let X be a random variable with finite mean μ and variance σ^2, then for any given positive ϵ we have:

$$Pr\left(|X-\mu|\leq\epsilon\right)\geq 1-\frac{\sigma^2}{\epsilon^2}=1-\frac{{\sigma_r}^2}{n\epsilon^2} \tag{4}$$

Where, X is the random variable, σ^2 is the variance $var(X)$, $\mu = E(X)$ represents the expectation of X, ϵ denotes the absolute error, ${\sigma_r}^2$ is the theoretical variance extract from real data, and n is the sample size. More specifically, if we want to be convinced that the probability of the average value lying inside the interval $(\mu-\epsilon,\ \mu+\epsilon)$ is no less than α (confidence greater than α). The minimum sample size (n) can be calculated by Equation 4 with given σ and ϵ. Take patients' LoS evaluation as an example, with the patient arrival model described in §3, 10% absolute error, 95% confidence, the minimum sample size as well as simulation time are shown in Table 8 by category of patients' acuity level. The statistical information (${\sigma_r}^2$) was retrieved from about 100,000 valid patient records in 2014. Note that the minimal simulation time is the minimum length of simulation required in order to satisfy the minimal sample size. This minimal simulation time in calculated by dividing the minimum sample size with average daily arrival of patients with that AL.

Table 8: Minimum sample size to evaluate patients' length of stay (LoS, Relative error: 10%, confidence: 95%.). The statistical information was retrieved from about 100,000 valid patient records in year 2014.

Items \ Acuity level	1	2	3	4	5
Mean (LoS, minute)	520.27	437.39	617.96	166.43	116.40
Standard deviation	493.88	734.57	806.79	182.055	94.24
Minimal sample size (number of patients)	901	2820	1704	1196	655
Percentage of patients arrival	0.55	7.10	31.00	49.30	12.05
Minimal simulation time (day)	410	99	13	6	13

It is clear to see from Table 8 that evaluating behavior of patients with AL 1 requires the longest simulation time because the proportion of patients arriving with AL 1 is fairly small (about 0.55 %). While for patients with AL 4, which makes up the largest proportion, the shortest simulation time is required. Thus, if the sample size of a given simulation scenario is greater than the minimum requirement, it is reasonable to simulate once without repetition. Otherwise, if the a given simulation scenario cannot generate enough sample size, repeating the same scenario with different random seeds is a must. The actual repetition times can be computed by dividing the minimum sample size requirement with the total arrival patients in the scenario separately for each AL and taking the maximum. For example, if we only want to evaluate patients' LoS in area B (AL 4 and 5) in a period of 7 days, we have to repeat the simulation for at least 2 times ($\lceil\frac{13}{7}\rceil$) with different random seed in order to generate enough samples. To evaluate other KPIs, e.g. occupancy of healthcare staff and medical test equipment, the same procedure can be used to compute the sample size requirements.

7. Simulation results and discussion

The ED model parameters used for characterizing agents' behavior have been calibrated based on real data from 2009 - 2011 [53]. This section illustrates the cross-validation results. That is, parameters for charactering the patient arrival model (described in §3) are retrieved from real data (in 2014), weekly arrival rates throughout the year are shown in Figure 4.

The ED resource configuration, which includes human resources (as well as their shifts), equipment, beds (as well as its layout) are specified the same as the real system. With patient arrival data shown in Figure 4 as input to specify the patient arrival pattern described in §3, we simulate to *imitate* the real operation in 2014. According to Table 8, two repetitions were applied to this scenario (the simulation scenario has 365 days). The simulation was carried out on an 8-node cluster with total number of 512 AMD Opteron™ Processor 6262 HE cores, and 2TB RAM. The whole simulation process takes about 30 minutes.

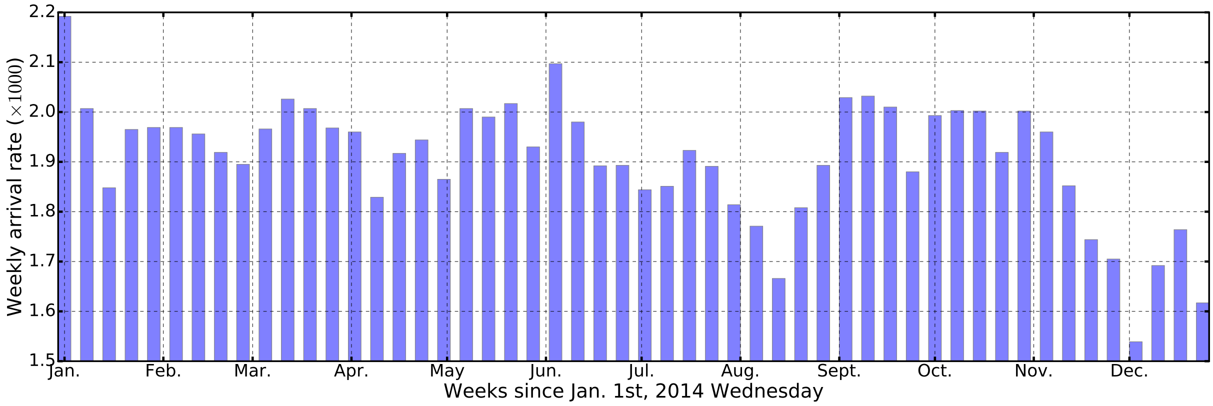
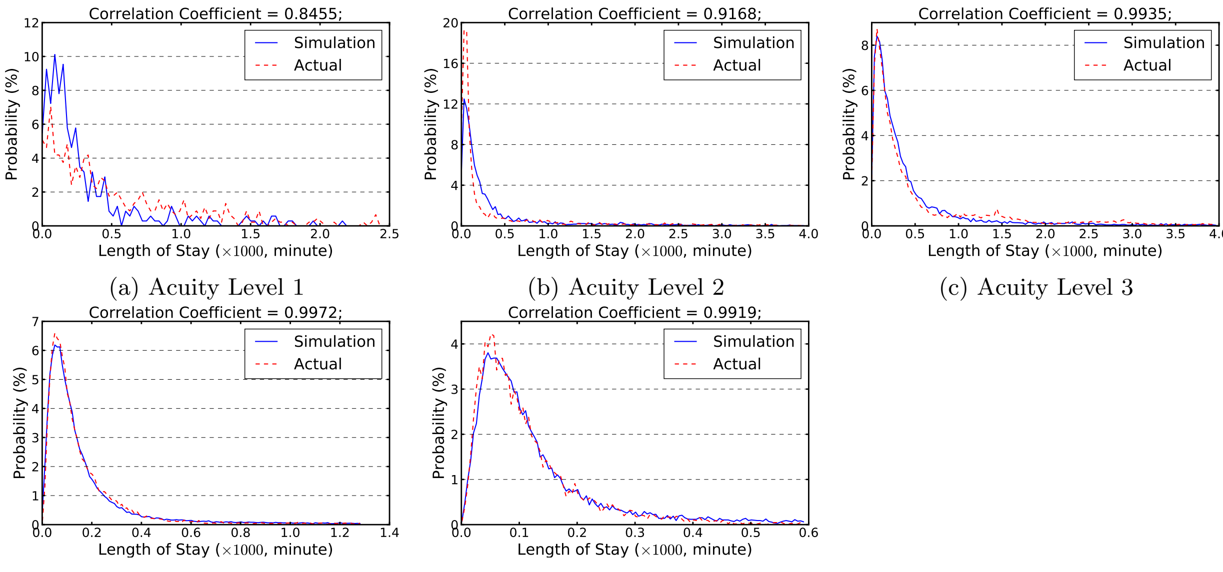


Figure 4: Patient weekly arrival rate, extracted from one-year actual data of the Hospital Hospital Taulì de Sabadell.

The patients' LoS, which can represent overall behavior of patients as well as EDs, was used as the KPI of the ED. When comparing simulated LoS with actual LoS, the absolute difference of their average cannot fully represent their differences because the same average may come from quite different distributions (e.g., a uniform distribution versus a normal distribution). Here we analyze the actual LoS distribution by using a histogram. Then we perform the same analysis for the simulated LoS. Thus, we will get two distributions and the similarity between them can be used as a metric to measure the simulation accuracy. The comparison results are shown in Figure 5, in which each figure represents the LoS distribution of patients with the same AL. The results on patient arrival were demonstrated previously in Figure 2a.



(a) Acuity Level 1

(b) Acuity Level 2

(c) Acuity Level 3

(d) Acuity Level 4

(e) Acuity Level 5

Figure 5: A set of simulation results about distribution of patients' LoS with each acuity level; each figure is the comparison of histogram of patients' LoS extracted from real data against simulation results. The statistical interval widths are: 30 minutes for acuity level 1, 2 and 3; 10 and 5 minutes for acuity level 4 and 5 respectively.

It can be seen from Figure 5 that simulation results are very similar to the actual ED behavior. However, as a result of the small number of AL 1 and 2 patients (about 0.55% and 7.1% respectively), the fitness for these two levels (Figure 5a and Figure 5b) are not as good as others. This is presumably because too less historical data about patients with AL 1 and 2 cannot enable us to extract sufficient information about their behavior.

8. Conclusion and future work

The ED is a complex, stochastic environment which has time-dependent behavior. Making changes to an ED's resources is a challenging activity since it has significant impact on its performance. Incorrect decisions may lead to serious consequences on the quality of service and cause unnecessary deaths. Simulation enables ED managers to make better decisions by letting them see the impact of changes before their implementation.

This article presents an agent-based model of EDs to study problems such as prolonged waiting times, inefficient use of ED resources, and unbalanced staff scheduling. With this model, policies such as staffing, human factors such as healthcare staff behavior and hypothetical events such as a flu outbreak can all be set up and their effects on system performance such as waiting time and throughput can be quantified [32, 33]. With the amount of adjustable parameters, the simulator is customizable to simulate a variety of scenarios. For example, the presented simulator is currently working as a platform to study MRSA transmission in EDs [29, 30] and as an experimental ED platform to provide data under various scenarios for knowledge discovery [31].

In summary, starting from simulating the EDs, our efforts proved the feasibility and ideality of using agent based modeling and simulation techniques to study healthcare systems. The cross-validation results showed that the developed ED simulator can accurately represent the emergent behavior of the complex ED system. Some demo application results previously presented in conferences proved that the simulator is ready to work as part of the decision support system (e.g., [32, 33]). The framework developed in our work is a step towards building a full model of an integrated care system. It opens a wide field of possible simulation scenarios for a better understanding of the integrated complex system [35]. The simulation is a crucial prerequisite for improving the coordination and integration of care and increasing the efficiency of resource allocation. As future work, the presented model will be calibrated for more EDs to validate its adaptability. More healthcare subsystems, such as EDs, out-patient service units, and in-patient units will all be simulated and connected together to allow for the assessment of ambulance and patient redirection policies. Additionally, a precise model of an ED is the cornerstone for further study, another work direction can be to combine the simulation model with optimization algorithms to optimize the performance of ED systems.

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